HALL EFFECT

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QUESTION TO BE INVESTIGATED

How to individual charge carriers behave in an external magnetic field that is perpendicular to their motion?

INTRODUCTION

The Hall effect is observed when a magnetic field is applied at right angles to a rectangular sample of material carrying an electric current. A voltage appears across the sample that is due to an electric field that is at right angles to both the current and the applied magnetic field. The Hall effect can be easily understood by looking at the Lorentz force on the current carrying electrons. The orientation of the fields and the sample are shown in Figure 1. An external voltage is applied to the crystal and creates an internal electric field (E_x). The electric field that causes the carriers to move through the conductive sample is called the drift field and is in the x-direction in Figure 1. The resultant drift current (J_x) flows in the x-direction in response to the drift field. The carriers move with an average velocity given by the balance between the force accelerating the charge and the viscous friction produced by the collisions (electrical resistance). The drift velocity appears in the cross product term of the Lorentz force as shown in equation 1. The transverse (y) component of the Lorentz force causes charge densities to accumulate on the transverse surfaces of the sample. Therefore, an electric field in the y-direction results that *just* balances the Lorentz force because there is no continuous current in the y-direction (Only a transient as the charge densities accumulate on the surface). The equilibrium potential difference between the transverse sides of the sample is called the Hall voltage.

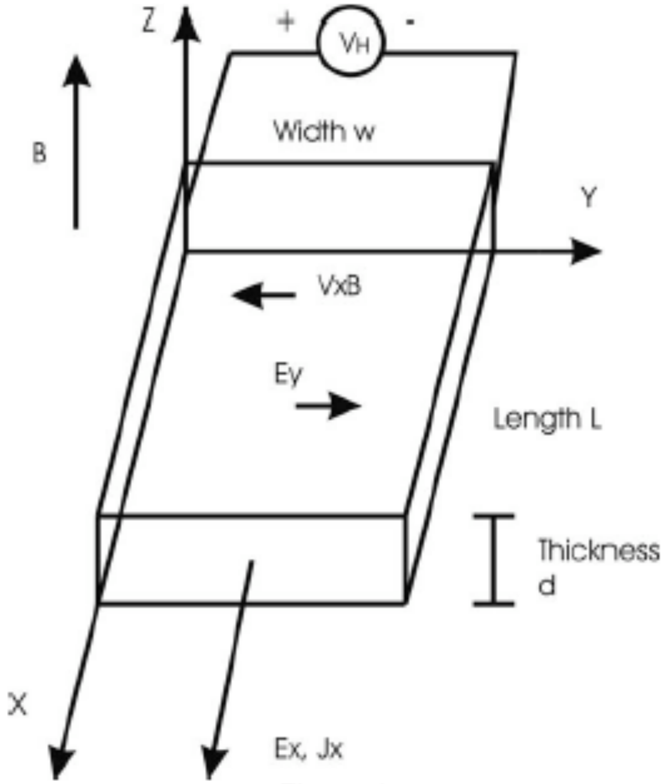


Figure 1
(Signs shown for the case of positive charge carriers)

A good measurement of the Hall voltage requires that there be no current in the y-direction. This means that the transverse voltage must be measured under a condition termed "no load". In the laboratory you can approximate the "no load" condition by using a very high input resistance voltmeter.

THEORY

The vector Lorentz force is given by:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{1}$$

where **F** is the force on the carriers of current, q is the charge of the current carriers, **E** is the electric field acting on the carriers, and **B** is the magnetic field inside the sample. The charge may be positive or negative depending on the material (conduction via electrons or "holes"). The applied electric field **E** is chosen to be in the x-direction. The motion of the carriers is specified by the drift velocity **v**. The magnetic field is chosen to be in the z-direction (Fig. 1).

The drift velocity is the result of the action of the electric field in the x-direction. The total current is the product of the current density and the sample's transverse area A ($I = J_xA$; $A = wt$). The drift current J_x is given by;

$$J_x = nqv_x \tag{2}$$

where n is the number density or concentration of carriers. The carrier density n is typically only a small fraction of the total density of electrons in the material. From your measurement of the Hall effect, you will measure the carrier density.

In the y-direction assuming a no load condition the free charges will move under the influence of the magnetic field to the boundaries creating an electric field in the y-direction that is sufficient to balance the magnetic force.

$$E_y = v_xB_z \tag{3}$$

The Hall voltage is the integral of the Hall field (E_y=E_H) across the sample width w.

$$V_H = E_Hw \tag{4}$$

In terms of the magnetic field and the current:

$$V_H = \frac{1}{nqd}BI = \frac{R_HBI}{d} \tag{5}$$

Here R_H is called the Hall coefficient. Your first task is to measure the Hall coefficient for your sample. This equation is where you will begin. The measurements you will take will be of V_H as a function of drift current I and as a function of magnetic field B. You will need to fix one parameter in order to intelligibly observe V_H. This experiment is setup to measure V_H as a function of I, so you will fix B for each trial you perform.

Other important and related parameters will also be determined in your experiment. The mobility μ is the magnitude of the carrier drift velocity per unit electric field and is defined by the relation:

$$v = \mu E \tag{6}$$

or,

$$\mu = \frac{v_x}{E_x} \tag{7}$$

This quantity can appear in the expression for the current density and its practical form.

$$J = \sigma E,\ or\ J_x = nqv_x = nq\mu E_x \tag{8}$$

We have the relations:

$$\sigma = nq\mu = \frac{1}{\rho} \tag{9}$$

Where σ is the conductivity and ρ is the resistivity. The total sample resistance to the drift current is:

$$R = \frac{\rho L}{wd} \tag{10}$$

Not only does the Hall coefficient give the concentration of carriers it gives the sign of their electric charge, by:

$$R_H = \frac{1}{nq} \tag{11}$$

The dimensions of the sample are w, the width which is to be oriented in the y-direction, d the thickness which is to be oriented in the z-direction which is perpendicular to the magnetic field and L, the length of the sample which is to be located along the x-direction. The sample should be about four times longer than it is wide so that the electric current streamlines have an opportunity to become laminar or the electric potential lines to become parallel and perpendicular to the edges of the sample. The Hall voltage should be zero when the sample is not in a magnetic field and the drift current is applied.

A plot of Hall voltage as a function of drift current at constant magnetic field will have a slope equal to $R_H B/d$. Thus, the slope multiplied by d/B is the magnitude of the Hall coefficient. Pay careful attention to the direction of fields and the sign of the voltages and obtain the sign of the charge carriers. For some materials the Hall coefficient is reasonably constant in the above equation and not a function of any of the experimental parameters. For some materials the Hall constant is a function of the magnetic field due to a magnetoresistance effect. Nevertheless, the Hall voltage is directly related to the magnetic field and the drift current, and it is inversely related to the thickness of the sample. The samples used for the measurement are made as thin as possible to produce the largest possible voltage for easy detection. The sample has no strength to resist bending and the probe is to be treated with great care. This applies to the probe of the Gaussmeter as well. Do not attempt to measure the thickness of the sample. The values for the dimensions of your sample are:

$w = 0.152$ cm, width,
$L = 0.381$ cm, length,
$d = 0.0152$ cm, thickness.

The Hall effect probe is a thin slab of indium arsenide, InAs, cemented to a piece of fiberglass. A four lead cable is attached so that the necessary electrical circuit can be used to detect the Hall voltage. The probe is equipped with a bakelite handle that is used to hold the probe in place. The white and green wires are used to measure the Hall voltage and the red and black wires carry the drift current.

EXPERIMENT

Before connecting the circuit, carefully measure the resistance of the current limiting resistor with an ohm meter (the resistance should be close to 100 ohms). Connect the probe into the electrical circuit shown in Figure 2 (See Appendix). You are to measure the Hall voltage, the drift current (determined from the voltage across the current limiting resistor and the known voltage across the system), and the drift voltage (this is the applied voltage minus voltage across the current limiting resistor). You have to keep track of directions and vectors.

Before you turn on the power, please make the following checks:

- Check the circuit carefully to ensure it makes sense to you.
- Put the meters on their correct scales. The voltage across the current limiting resistor will be less than 1 Volt. The Hall voltage will be even less.

As the drift current is stepped up by the computer, check the following:

- The drift current must not exceed 10ma this corresponds to 1.0 volt across the 100 ohm current limiting resistor (This voltage is shown on one of the voltmeters in Figure 2). **Conduct all your scans between 0V and 1V.**
- The probe should remain cool to the touch. If it warms at all something is drastically wrong! Turn off the power and disconnect the Hall probe immediately.

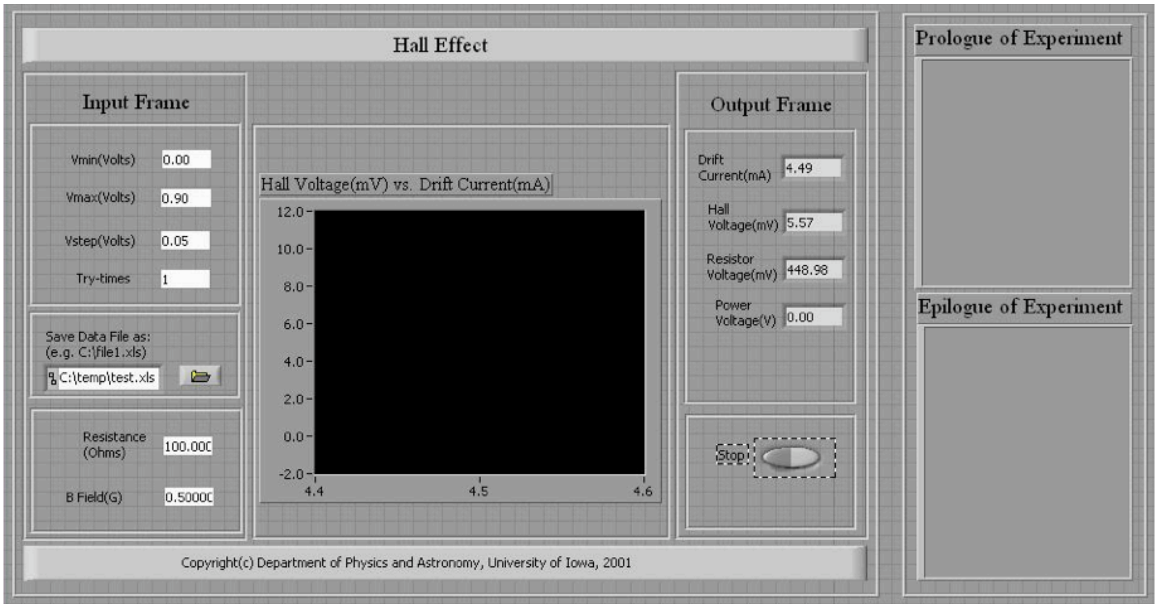
Magnet Calibration:

The magnetic field B is produced by an electromagnet so the field strength is proportional to the current through its coil. There is a Gaussmeter (which also uses the Hall effect) for you to directly measure the magnetic field strength. Note that the probe for the Gaussmeter is sensitive to the vector component of B that is normal to the surface of the probe.

V_H depends on the magnitude of the B field and the drift current I. You will be varying I and will want to make B a constant for each trial. Therefore, you will want to understand how the magnet power supply current I_M relates to the magnitude of the B field. Use the Cenco Gaussmeter, mentioned above, to construct a calibration curve for the electromagnet (B vs. I_M). Be certain that the Gaussmeter is rotated to produce the maximum reading possible. Fit the calibration curve to a function. Use the instrumental values to determine the uncertainty in B at each current setting. In the next section the calibration curve will be used to determine B for each measured magnet current.

Determine R_H:

You will use LabView to control the drift current in the Hall sample. A few things must be in order to have the VI operate correctly, see Figure 3.



Vmin(Volts) 0.00
Vmax(Volts) 0.90
Vstep(Volts) 0.05
Try-times 1
Save Data File as: (e.g. C:\file1.xls) C:\temp\test.xls
Resistance (Ohms) 100.000
B Field(G) 0.50000
Drift Current(mA) 4.49
Hall Voltage(mV) 5.57
Resistor Voltage(mV) 448.98
Power Voltage(V) 0.00

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Figure3. The VI. NOTE: In order to operate, the "stop button" must be red, the file path of your output file must not exceed the length of the field provided and must contain the file path "**C:\temp***.xls" where "*" is your file, the voltage must never exceed 1V, and "try times" should be set to 1. The VI will automatically calculate the drift current from the measured resistor voltage V_R and the Resistance value that you input. The output file will contain all of the raw data, thankfully.

In addition to controlling the drift current, the VI will also record data from the two volt meters (see Appendix). One of the meters will measure the voltage across the resistor in the drift circuit (bottom), and the other will measure the voltage across the y-direction of the probe (top; the hall voltage!). As the computer steps up the driving voltage (and therefore drift current) it will record the value from each of these meters, and then output them when the program finishes. The probe is InAs, which has a nearly constant Hall coefficient so, from Eq. (5), your data should be very close to a straight line (V_H vs. I) with a slope that is related to the Hall constant R_H.

Run the LabView program for 10 equally spaced values of the magnet currents. You may wish to span the entire range of I_M, or choose the most linear portion of your calibration to reside in. Save the data curves for each run and the magnet currents for later analysis. This will allow you to construct a family of curves that represent V_H vs. I at constant B. From the magnetic field and the fitted slope you will be able to determine a Hall coefficient R_H for each curve. Should all of the slopes appear similar? How does changing B affect the behavior of Eq. (5)? Histogram the values for R_H and determine a global R_H and its uncertainty.

You should report the magnitude of the Hall coefficient and the sign of the charge carriers. To do this you need to know the direction of the magnetic field. This is most easily found by using a compass. Using this evidence make a claim about the behavior of moving charge carriers within a transverse magnetic field.

The slope of each of your curves is related to the Hall coefficient. The intercept is related to the residual field and your zero field Hall Voltage. Give the magnitude of the intercept and offer an explanation of its origin. Can the data be credibly fit to a one-parameter line (intercept assumed equal to zero)?

Find (and report) the ratio of the number of carriers per unit volume to the number of atoms of InAs per unit volume.

Report your value of the mobility of the charge carriers and the conductivity of the sample. Is the mobility of the carriers a function of the magnetic field? Support your claim with statistical evidence. Indium Arsenide is used as a probe in Hall effect Gaussmeters because the mobility and conductivity and hence these coefficients are not strongly a function of the magnetic field. Note, in the experiment you do not measure the resistance of the sample directly. To get the resistance you will need to determine the driving voltage (that drives the drift current) and divide this by the drift current. You have in your measurements the voltage applied by the computer and the voltage across the 100 ohm resistor. Use the measured value of the resistance of the resistor and use the actual value in your calculations of current and then in the calculation of the sample resistance.

Plot the conductivity and the mobility as a function of the magnetic field. You should find a very small effect, if any, when you fit the mobility, conductivity or Hall coefficient as a function of magnetic field. To show this effect, calculate the difference between the measured values and the average value. These values are called residuals. Do the residuals vary as a function of magnetic field? Can you quantify a systematic effect, given the errors?

B. Studying the Electromagnet

Using the global Hall coefficient for your newly calibrated Hall probe, you can now use it as an instrument to measure the spatial variation of the magnetic field of your electromagnet.

Measure the Hall voltage as a function of distance from the center of the pole pieces to about a meter away from the center for a 5mA constant drift current and constant magnet current. Move the probe away from the magnet in a direction perpendicular to the magnetic field that is nearly constant at the center and decreasing sharply at the edge of the pole piece. Use small steps when you are close to the magnet and larger steps when outside. The field changes rapidly inside the pole pieces and you will want greater resolution there. A simple function will not fit the dependence in this region because of the complexities of magnetic field fringing at the edge. A short distance outside the edge of the pole pieces and to a distant point, a functional fit to the data should permit you to compare the actual field dependence on distance to a model. What dependence would you expect to see?

Now, double the separation between the magnet pole pieces and repeat the measurements. Does the field at the center change by roughly a factor of 2? Support this claim with statistical evidence. Report your results by plotting your data as a function of distance from the center of the magnet. Finally, move the pole pieces to maximum separation and measure the field from the center of the left pole piece to the center of the right pole piece in about 10 equal steps. Is the field constant? If not, why not? Present your results as a plot.

Nominal Electrical Characteristics (From the manufacturer) of InAs Hall Probe:

Internal Resistance (Ohms)	1
Hall Constant, minimum (m^3/C)	0.0001
Hall Null Voltage (Volts)	0.01
Flux Density Range (Tesla)	0-1
Load Resistance for maximum linearity (Ohms)	10
Load Resistance for maximum power transfer (Ohms)	2
Frequency response (MHz)	1

COMMENTS:

Because the Hall coefficient of a material is a function of the material and the impurity doping level you cannot find a “standard” textbook or handbook value for the Hall coefficient for the material in the Cenco probe. Note that the Hall coefficient is best reported in meters cubed per coulomb (SI units). Unfortunately, it is usually reported in the units (cm^3/C).

InAs has a relatively small band gap so the carrier density should be roughly the intrinsic carrier density. These carriers are produced by the thermal excitation of the electrons from the valence band into the conduction band. You can estimate this as the density of valence electrons ($\sim 7 \times 10^{28}$ m^{-3}) multiplied by the Boltzmann factor, $\exp[-E_g/kT]$, where E_g is about 0.35 eV for InAs and kT is room temperature (which is about 1/40 eV). Does this agree with your measurement? For additional information see Melissinos §7.5 p 283.

APPENDIX:

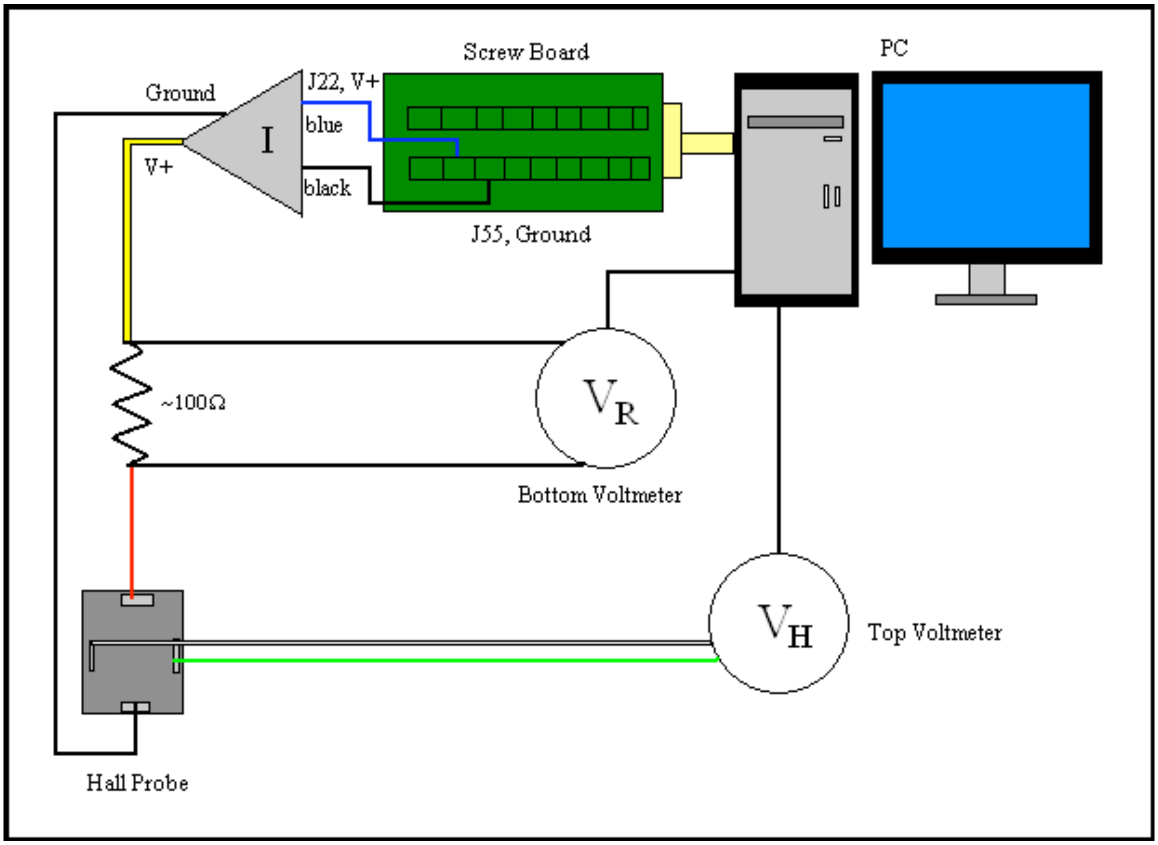

Figure 2. Schematic of the Experimental Setup